EJERCICIO (19:04)

Calcular:

$$\boldsymbol{\beta_{pk} \equiv -\left(f_k^*\,,h_p\right)}$$

$$f_k = \frac{1}{\sqrt{4\pi k}}\ e^{ike^{X-T}}$$

$$h_p = \frac{1}{\sqrt{4\pi p}}\ e^{-ip(T-X)}$$

$$\partial_0 f_k = \frac{1}{\sqrt{4\pi k}} ike^{X-T}(-1)\ e^{ike^{X-T}} = -i\ k\ e^{X-T} f_k$$

$$\partial_0 h_p = \frac{1}{\sqrt{4\pi p}} (-i\ p) e^{-ip(T-X)} = -i\ p\ h_p$$

$$(A\,,B) \equiv i\int_{-\infty}^{\infty} dX\ (A^*\partial_0 B - B\ \partial_0 A^*)$$

$$\left(f_k^*\,,h_p\right) = \ i\int_{-\infty}^{\infty} dX\ \left(f_k \partial_0 h_p - h_p\ \partial_0 f_k\right)$$

$$\left(f_k^*\,,h_p\right) = \ i\int_{-\infty}^{\infty} dX\ \left(f_k\left(-i\ p\ h_p\right) - h_p(-i\ k\ e^{X-T} f_k)\right)$$

$$\left(f_k^*\,,h_p\right) = \int_{-\infty}^{\infty} dX\ \left(f_k\left(\ p\ h_p\right) - h_p(\ k\ e^{X-T} f_k)\right)\ = \int_{-\infty}^{\infty} dX\ (p - k\ e^{X-T}) f_k\ h_p$$

$$\left(f_k^*\,,h_p\right) = \int_{-\infty}^{\infty} dX\ (p - k\ e^{X-T}) \frac{1}{\sqrt{4\pi k}}\ e^{ike^{X-T}}\ \frac{1}{\sqrt{4\pi p}}\ e^{-ip(T-X)}$$

Como la integral es invariable en el tiempo se calcula para T = 0

$$\left(f_k^*\,,h_p\right) = \frac{1}{4\pi\ \sqrt{p\ k}} \int_{-\infty}^{\infty} dX\ (p - k\ e^X)\ e^{ike^X} e^{ipX}$$

$$\left(f_k^*\,,h_p\right) = \frac{1}{4\pi\ \sqrt{p\ k}} \int_{-\infty}^{\infty} dX\ (p - k\ e^X)\ e^{i\left(ke^X + pX\right)}$$

$$\boxed{\boldsymbol{\beta_{pk} \equiv -\frac{1}{4\pi\ \sqrt{p\ k}} \int_{-\infty}^{\infty} dX\ (p - k\ e^X)\ e^{i\left(ke^X + pX\right)}}}$$